D–6461

Sub. Code
31111

DISTANCE EDUCATION

M.Sc.(Mathematics) DEGREE EXAMINATION,
DECEMBER 2024.

First Semester

ALGEBRA — I

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

SECTION A — ($10 \times 2 = 20$ marks)

Answer ALL the questions.

1. Define onto mapping. Give an example.

2. If H is the only subgroup of order $o(H)$ of a group G, prove that H is a normal subgroup of G.

3. Define group isomorphism.

4. State the first part of Sylow's theorem.

5. Show that the group of order 21 is not simple.

6. Define commutative ring with an example.

7. If I is an ideal of a ring R containing the unit element, show that $I = R$.

8. If R is a ring and $a \in R$, let $r(a) = \{x \in R / ax = 0\}$, prove that $r(a)$ is a right – ideal of R.

9. Let R be an Euclidean domain. Suppose that $a,b,c \in R$, $a \mid bc$ but $(a,b) = 1$. Prove that a / c.

10. Prove that an Euclidean ring posses an unit element.

SECTION B — (5 × 5 = 25 marks)

Answer ALL the questions, choosing either (a) or (b).

11. (a) Prove that there is a one-to-one correspondence between the set of integers and the set of rational numbers.

Or

(b) State and prove Euler's theorem.

12. (a) If p is a prime numbers and $p / o(G)$, then prove that G has an element of order p.

Or

(b) Show that any two p-sylow subgroups of a group G are conjugate.

13. (a) Prove that any finite integral domain is a field.

Or

(b) If D is an integral domain and D is of finite characteristic, prove that the characteristics of D is a prime number.

14. (a) Let R be a commutative ring with unit element whose only ideals are (O) and R itself. Prove that R is a field.

Or

(b) State and prove the division algorithm.

15. (a) Prove that $J[i]$ is an Euclidean ring.

Or

(b) If R is an integral domain, then prove that $R[x_1, x_2, ..., x_n]$ is also an integral domain.

SECTION C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Prove that every group is isomorphic to a subgroup of $A(S)$ for some appropriate S.

17. Let G be a group and suppose that G is the internal direct product of $N_1, ... N_n$. Let $T = N_1 \times N_2 \times \cdots \times N_n$. Then prove that G and T are isomorphic.

18. State and prove Fermat's theorem.

19. Let $R = \{(a,b) / a,b \in R\}$ and the operation addition and multiplication are defined as $(a,b) + (c,d) = (a + c, b + d)$ and $(a,b) \cdot (c,d) = (ac - bd, ad + bc)$. Show that R is a field.

20. (a) State and prove Gauss lemma.

(b) State and prove the unique factorization theorem.

D–6462

Sub. Code
31112

DISTANCE EDUCATION

M.Sc.(Mathematics) DEGREE EXAMINATION,
DECEMBER 2024.

First Semester

ANALYSIS — I

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL the questions.

1. Define ordered set. Give an example.

2. Define a metric space.

3. Balls are convex. Justify.

4. Define compact set. Give an example.

5. Define convergence sequence. Give an example.

6. If $p > 0$, then prove that $\lim_{n\to\infty} \sqrt[n]{p} = 1$.

7. Define monotonic function.

8. Define open balls in R^n.

9. State the intermediate value theorem.

10. Define contraction.

SECTION B — $(5 \times 5 = 25$ marks)

Answer ALL the questions, choosing either (a) or (b).

11. (a) If $x \in R$, $y \in R$, and $x > 0$, then prove that there is a positive integer n such that $nx > y$.

Or

(b) Prove that there is not rational number whose square is 12.

12. (a) Let A be the set of all sequences whose elements are the digits 0 and 1. Prove that the set A is uncountable.

Or

(b) Prove that compact subsets of metric spaces are closed.

13. (a) If $\overline{E}$ is the closure of a set E in a metric space X, then prove that $diam\overline{E} = diamE$.

Or

(b) If f is a continuous mapping of a metric space X into a metric space Y and if E is a connected subset of X, then prove that $f(E)$ is connected.

14. (a) Let f be a continuous real valued function on a metric space X. Let $Z(f)$ be the set of all $p \in X$ at which $f(p) = 0$. Prove that $Z(f)$ is closed.

Or

(b) Prove that a uniformly continuous function of a uniformly continuous function is uniformly continuous.

15. (a) State and prove Rolle's theorem.

Or

(b) State and prove mean value theorem.

SECTION C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Let p be a non-empty perfect set in R^k. Prove that p is uncountable.

17. Prove that $\Sigma \frac{1}{n^p}$ converges if $p > 1$ and diverges if $p \leq 1$.

18. State and prove the root test.

19. State and prove the Bolzano – Weierstrass theorem.

20. State and prove the implicit function theorem.

D–6463

Sub. Code
31113

DISTANCE EDUCATION

M.Sc.(Mathematics) DEGREE EXAMINATION,
DECEMBER 2024.

First Semester

ORDINARY DIFFERENTIAL EQUATIONS

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours — Maximum : 75 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL the questions.

1. Verify that $\phi_1(x) = e^x, x > 0$, is a solution of $xy'' - (x+1)y' + y = 0$.

2. Find all solutions of $y'' = 0$.

3. State the uniqueness theorem for linear equation with constant coefficient $L(y) = 0$.

4. Write down the Chebyshev equation.

5. Find the value of $P_n(1)$ and $P_n(-1)$.

6. Find the regular singular points for $(1-x^2)y'' - 2xy' + 2y = 0$.

7. Show that the solution ϕ of $y^1 = y^2$ which passes through the point (x_0, y_0) is given by $\phi(x) = \dfrac{y_0}{1 - y_0(x - x_0)}$.

8. Write the Bessel function of zero order of the first kind.

9. Consider the initial value problem $y'_1 = y_2^2 + 1 \quad y'_2 = y_1^2$, $y_1(0) = 0,\ y_2(0) = 0$. Compute the first three successive approximations ϕ_0, ϕ_1, ϕ_2.

10. State the non-local existence theorem for $y' = f(x, y),\ y(x_0) = y_0,\ |y_0| < \infty$.

SECTION B — (5 × 5 = 25 marks)

Answer ALL the questions, choosing either (a) or (b).

11. (a) Find all the solution of $y'' + 9y = \sin 3x$.

Or

(b) If ϕ_1, ϕ_2 are two solutions of $L(y) = 0$ on an interval I containing a point x_0, then prove $W(\phi_1, \phi_2)(x) = e^{-\alpha(x - x_0)} W(\phi_1, \phi_2)(x_0)$.

12. (a) Let $\alpha_1, \alpha_2, \ldots, \alpha_n$ be any n constants, and let x_0 be any real number. Prove that there exists a solution ϕ of $L(y) = 0$ on $-\infty < x < \infty$ satisfying $\phi(x_0) = \alpha_1,\ \phi'(x_0) = \alpha_2, \ldots, \phi^{(n-1)}(x_0) = \alpha_n$.

Or

(b) Show that $\int_{-1}^{1} P_n(x) P_m(x)\, dx = 0$ when $n \neq m$.

13. (a) Find all solutions of the equation $x^2 y'' + x y' + y = 0$.

Or

(b) Show that $x^{1/2} J_{1/2}(x) = \dfrac{\sqrt{2}}{\sqrt{\left(\frac{1}{2}\right)}} \sin x$.

14. (a) Find all real valued solutions of the equation $y' = \dfrac{x + x^2}{y - y^2}$.

Or

(b) Compute the first four successive approximation $\phi_0, \phi_1, \phi_2, \phi_3$ for the equation $y' = 1 + xy,\ y(0) = 1$.

15. (a) Let f be continuous and satisfy Lipschitz condition on R. If ϕ and ψ are two solutions of $y' = f(x, y)$, $y(x_0) = y_0$ on an interval I containing x_0, then prove that $\phi(x) = \psi(x)$ for all x in I.

Or

(b) Let f be a continuous vector-valued function defined on $R: |x - x_0| \le a,\ |y - y_0| \le b,\ (a, b > 0)$ and f satisfies Lipschitz condition on R and K is a Lipschitz constant for f in R, then prove that $|\phi(x) - \phi_K(x)| \le \dfrac{M(K_\alpha)^{K+1}}{K(K+1)!} e^{K_\alpha}$.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Let ϕ be any solution of $L(y) = y^{(n)} + a_1 y^{(n-1)} + \cdots + a_n y = 0$ on an interval I containing a point x_0. Prove that for all x in I $\|\phi(x_0)\| e^{-K|x - x_0|} \le \|\phi(x)\| \le \|\phi(x_0)\| e^{K|x - x_0|}$, where $K = 1 + |\alpha_1| + \cdots + |\alpha_n|$.

17. Solve $(1 - x^2) y'' - 2x + y' + \alpha(\alpha + 1) y = 0$.

18. Derive Bessel function of order α of the first kind $J_\alpha(x)$.

19. Find the singular points of the following, and determine those which are regular singular points : (a) $3x^2y''+x^6y'+2xy=0$ (b) $xy''+4y=0$.

20. Let f,g be continuous on R, and suppose f satisfies a Lipschitz condition with Lipschitz constant K. Let ϕ and ψ be solutions of $y'=f(x,y)$, $y(x_0)=y_1$; $y^1=g(x,y)$, $y(x_0)=y_2$ respectively on an interval z containing x_0 and contained in R. If the inequalities $|f(x,y)-g(x,y)|\leq\in$, $((x,y)\in R)$; $|y_1-y_2|\leq\delta$ are valid then prove that $|\phi(x)-\psi(x)|\leq\delta e^{K|x-x_0|}+\frac{E}{K}\left(e^{K|x-x_0|}-1\right)$ for all x in Z.

D–6464

Sub. Code
31114

DISTANCE EDUCATION

M.Sc.(Mathematics) DEGREE EXAMINATION,
DECEMBER 2024.

First Semester

TOPOLOGY — I

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

SECTION A — ($10 \times 2 = 20$ marks)

Answer ALL the questions.

1. Define Cartesian product of two sets.

2. Define countable set. Give an example.

3. Define basis for a topology.

4. What is meant by the subspace topology?

5. Define the quotient topology. Give an example.

6. When will you say that two topological spaces are continuous?

7. Define component and path component.

8. Define second countability axiom.

9. Define Hausdorff space.

10. What is meant by completely regular space?

SECTION B — ($5 \times 5 = 25$ marks)

Answer ALL the questions, choosing either (a) or (b).

11. (a) Prove that a finite product of countable sets is countable.

Or

(b) Prove that there exists an uncountable well – ordered set, every section of which is countable.

12. (a) If $\{\tau_\alpha\}$ is a collection of topologies on X, show that $\cap \tau_\alpha$ is a topology on X. Is $\cup \tau_\alpha$ a topology on X?

Or

(b) Let X be a set; let $\mathcal{B}$ be a basis for a topology τ on X. Prove that τ equals the collection of all unions of elements of $\mathcal{B}$.

13. (a) Let Y be a subspace of X; let A be a subset of Y; let $\overline{A}$ denote the closure of A in X. Prove that the closure of A in Y equals $\overline{A} \cap Y$.

Or

(b) State and prove the pasting lemma.

14. (a) Prove that the image of a connected space under a continuous map is connected.

Or

(b) Show that the rationals Q are not locally compact.

15. (a) Show that if X has a countable basis, every collection of disjoint open sets in X is countable.

Or

(b) Prove that every metrizable space is normal.

SECTION C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Prove that the set $\mathbb{Z} \times \mathbb{Z}_+$ is countably infinite.

17. Let $\bar{d}(a,b) = \min\{|a-b|, 1\}$ be the standard bounded metric on $\mathbb{R}$. If x and y are two points of $\mathbb{R}^w$, define $D(x,y) = l.u.b. \left\{ \frac{\bar{d}(x_1, y_1)}{i} \right\}$. Prove that D is a metric that induces the product topology on $\mathbb{R}^w$.

18. State and prove the sequence lemma.

19. State and prove the tube lemma.

20. State and prove the Urysohn metrization theorem.

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D–6465

Sub. Code
31121

DISTANCE EDUCATION

M.Sc.(Mathematics) DEGREE EXAMINATION,
DECEMBER 2024.

Second Semester

ALGEBRA — II

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

SECTION A — ($10 \times 2 = 20$ marks)

Answer ALL the questions.

1. Define vector space.

2. Define internal direct sum.

3. Define dual space.

4. Define norm.

5. Define group automorphism.

6. Define normal extension.

7. Define invariant of linear transformation.

8. Define trace of a matrix.

9. Define Hermition and skew Hermition of a matrix.

10. If $TT^* = 1$ then prove that $T \in A(V)$ is unitary.

SECTION B — (5 × 5 = 25 marks)

Answer ALL the questions, choosing either (a) or (b).

11. (a) Prove that the Kernal of a homomorphism is a subspace.

Or

(b) Prove that if $v_1,...,v_n$ are in V then either they are linearly independent or some v_k is a linear combination of the preceding ones, $v_1,...,v_{k-1}$.

12. (a) If $S,T \in Hom(V,W)$ and $v_iS = v_iT$ for all elements v_i of a basis of V, prove that $S = T$.

Or

(b) Prove that W^1 is a subspace of a vector space V.

13. (a) Prove that it is impossible by straightedge and compass alone, to trisect 30°.

Or

(b) Prove that the fixed field of G is a subfield of K.

14. (a) Prove that if $u \in V_1$ is such that $uT^{n_1-k} = 0$, where $0 < k \leq n$, then $u = u_0T^k$ for some $u_0 \in V_1$.

Or

(b) Prove that $t_r(A+B) = t_r(A) + t_r(B)$.

15. (a) Prove that if $T \in A(V)$ is such that $(vT,v) = 0$ for all $v \in V$, then $T = 0$.

Or

(b) Let K be a field and let G be a finite subgroup of the multiplicative group of non zero elements of K. Then prove that G is a cyclic group.

SECTION C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Let V be a finite – dimensional inner product space; then prove that V has an orthonormal set as a basis.

17. Prove that the element $a \in K$ is algebraic over F if and only if $F(a)$ is a finite extension of F.

18. Let K be a normal extension of F and let H be a subgroup of $G(K,F)$; let $K_H = \{x \in K / \sigma(x)$ for all $\sigma \in H\}$ be the fixed field of H. Then prove that

    (a) $[K : K_H] = O(H)$.

    (b) $H = G(K, K_H)$.

19. Prove that the elements S and T in $A_F(V)$ are similar in $A_F(V)$ if and only if they have the same elementary divisors.

20. Prove that for every prime number p and every positive integer m there is a unique field having p^m elements.

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D–6466

Sub. Code
31122

DISTANCE EDUCATION

M.Sc.(Mathematics) DEGREE EXAMINATION, DECEMBER 2024.

Second Semester

ANALYSIS – II

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours   Maximum : 75 marks

SECTION A — ($10 \times 2 = 20$ marks)

Answer ALL the questions.

1. Define Riemann integral.

2. Define refinement.

3. Suppose $f \geq 0$, f is continuous on $[a,b]$ and $\int_a^b f(x)\,dx = 0$. Prove that $f(x) = 0$.

4. Let $f_n(x) = \dfrac{x^2}{x^2 + (1-nx)^2}, 0 \leq x \leq 1, n = 1,2,...$ Show that $\{f_n\}$ is uniformly bounded.

5. Define orthogonal system of functions.

6. Prove that $\lim\limits_{x \to 0} \dfrac{\log(1+x)}{x} = 1$.

7. Define outer measure.

8. Define Borel set.

9. Define measurable function.

10. Define Cauchy sequence in $L^2(\mu)$.

SECTION B — ($5 \times 5 = 25$ marks)

Answer ALL the questions, choosing either (a) or (b).

11. (a) Prove that $\int_{-a}^{b} f d\alpha \leq \int_{a}^{-b} f\, d\alpha$.

Or

(b) State and prove the fundamental theorem of calculus.

12. (a) If $f \in R(\alpha)$ on $[a,b]$ and if $a < c < b$, then prove that $f \in R(\alpha)$ on $[a,c]$ and on $[c,b]$ and $\int_{a}^{c} f\, d\alpha + \int_{c}^{b} f\, d\alpha = \int_{a}^{b} f\, d\alpha$.

Or

(b) Suppose $\{f_n\}$ is a sequence of functions defined on E, and suppose $|f_n(x)| \leq M_n$, $x \in E$, $n = 1,2,....$ Prove that Σf_n converges uniformly on E if $\sum M_n$ converges.

13. (a) Let $\mathcal{B}$ be the uniform closure of an algebra $\mathcal{A}$ of bounded functions. Prove that $\mathcal{B}$ is a uniformly closed algebra.

Or

(b) If, for some x, there are constants $\delta > 0$ and $M < \infty$ such that $|f(x+t) - f(x)| \leq M|t|$ for all $t \in (-\delta, \delta)$, then prove that $\lim_{N \to \infty} S_n(f; x) = f(x)$.

14. (a) If $E = \bigcup_{n=1}^{\infty} E_n$, then prove that $\mu^*(E) \le \sum_{n=1}^{\infty} \mu^*(E_n)$.

Or

(b) Prove that if f is one-to-one continuous mapping of $\mathbb{R}$ onto $\mathbb{R}$, then f maps Borel sets onto Borel sets.

15. (a) Let f be measurable and non-negative on X. For $A \in m$, $\phi(A) = \int_A f \, d\mu$. Prove that ϕ is countably additive on m.

Or

(b) State and prove Fatou's theorem.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Prove that the sequence of functions $\{f_n\}$, defined on E, converges uniformly on E if and only if for every $\in > 0$ there exists an integer N such that $m \ge N$, $n \ge N, x \in E$ implies $|f_n(x) - f_m(x)| \le \in$.

17. State and prove the Stone – Weierstrass theorem.

18. If $x > 0$ and $y > 0$, then prove that the beta function $B(x, y)$ is $\int_0^1 t^{x-1}(1-t)^{y-1} dt = \frac{\Gamma(x) \ \Gamma(y)}{\Gamma(x+y)}$.

19. State and prove Egoroff's theorem.

20. State and prove Lebesgue's monotone convergence theorem.

D–6467

Sub. Code
31123

DISTANCE EDUCATION

M.Sc.(Mathematics) DEGREE EXAMINATION,
DECEMBER 2024.

Second Semester

TOPOLOGY — II

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours   Maximum : 75 marks

SECTION A — ($10 \times 2 = 20$ marks)

Answer ALL the questions.

1. Define locally metrizable space.

2. What is countable intersection condition?

3. Define compactification of a space.

4. Define refinement.

5. Define Lindelof space.

6. Define completion of a metric space.

7. Define totally bounded space.

8. Show that if the subset $\Im$ of $C(X,Y)$ is equicontinuous then so is its closure.

9. Define finite dimensional space.

10. Show that the cantor set has dimension 0.

SECTION B — ($5 \times 5 = 25$ marks)

Answer ALL the questions, choosing either (a) or (b).

11. (a) Let A and B be closed disjoint subsets of the normal space X. Prove that there exists a continuous function $f : X \to [0, 1]$ such that $f^{-1}(\{0\}) = A$ and $f(B) = \{1\}$ if and only if A is a G_σ set in X.

Or

(b) Prove that a product of completely regular space is completely regular.

12. (a) Let Q be a locally finite collection of subsets of X. Prove that the collection $B = \{\overline{A}\}_{A \in Q}$ of the closures of the elements of Q is locally finite.

Or

(b) Let X be a metrizable space. Prove that X has a basis that is countably locally finite.

13. (a) Prove that every closed subspace of paracompact space is paracompact.

Or

(b) Let X be a topological space. Prove that the set $B(X, \mathbb{R})$ of all bounded functions $f : X \to \mathbb{R}$ is complete under the sup.metric δ.

14. (a) Show that in the compact open topology, $C(X, Y)$ is Hausdorff if Y is Hausdorff.

Or

(b) If X is a compact Hausdorff space, or a complete metric space then prove that X is a Baire space.

15. (a) Prove that if Y is a closed subset of X, and if X has finite dimension then so does Y, and $\dim Y \leq \dim X$.

Or

(b) Prove that any compact subset C of $\mathbb{R}^2$ has topological dimension at most 2.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. State and prove the Tietze extension theorem.

17. Let X be a regular space with a basis B that is countably locally finite. Prove that X s metrizable.

18. State and prove Peano space filling curve theorem.

19. Let $h:[0,1] \to \mathbb{R}$ be a continuous function. Prove that, given $\in > 0$, there is a function $g:[0,1] \to R$ with $|h(x) - g(x)| < \in$ for all x, such that g is continuous and nowhere differentiable.

20. Prove that every compact subset of R^n has topological dimension at most N.

D–6468

Sub. Code
31124

DISTANCE EDUCATION

M.Sc.(Mathematics) DEGREE EXAMINATION,
DECEMBER 2024.

Second Semester

PARTIAL DIFFERENTIAL EQUATIONS

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours   Maximum : 75 marks

SECTION A — ($10 \times 2 = 20$ marks)

Answer ALL the questions.

1. Write the Lipschitz condition.

2. Define orthogonal trajectories.

3. Define an integrating factor.

4. Eliminate a and b from $z = (x + a)(y + b)$.

5. Define complete integral.

6. Define singular integral.

7. State Poisson's equation.

8. Write the one dimensional wave equation.

9. Write the exterior Neuman problem.

10. State the inlerior Dirichelt problem.

SECTION B — ($5 \times 5 = 25$ marks)

Answer ALL the questions, choosing either (a) or (b).

11. (a) Show that the direction cosines of the tangent at the point (x, y, z) to the conic $ax^2 + by^2 + cz^2 = 1$, $x + y + z = 1$ are proportional to $(by - cz, cz - ax, ax - by)$.

Or

(b) Find the integral curves of the set of equation :

$$\frac{dx}{x(y-z)} = \frac{dy}{y(z-x)} = \frac{dz}{z(x-y)}.$$

12. (a) Verify that the differential equation $(y^2 + yz)\,dx + (xz + z^2)\,dy + (y^2 - xy)\,dz = 0$ is integrable and find its primitive.

Or

(b) Verify that the following equation is integrable and find their primitives $yz\,dx + xz\,dy + xy\,dz = 0$.

13. (a) Eliminate the arbitrary function f from the equation $z = x + y + f(xy)$.

Or

(b) Find the general integrals of the linear partial differential equation

$$px(x + y) = qy(x + y) - (x - y)(2x + 2y + z).$$

14. (a) Verify that the partial differential equation $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = \frac{2z}{x}$ is satisfied by $z = \frac{1}{x}\phi(y-x) + \phi'(y-x)$ where ϕ is an arbitrary function.

Or

(b) Find a particular integral of the equation $(D^2 - D')z = e^{2x+y}$.

15. (a) If $p > 0$ and $\psi(r)$ is given by equation $\psi(r) = \int_v \frac{p(r')\,dr'}{|r - r'|}$, where the volume V is bounded, prove that $\lim_{r\to\infty} r\psi(r) = M$ where $M = \int_v p(r')dr'$.

Or

(b) Derive the d'Alembert's solution of one dimensional wave equation.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Solve the equations $\frac{dx}{y + \alpha z} = \frac{dy}{z + \beta x} = \frac{dz}{x + ry}$.

17. Verify that the equation $yz(y+z)dx + xz(x+z)dy + xy(x+y)dz = 0$ is integrable and find its solution.

18. Show that the only integral surface of the equation $2q(z - px - qy) = 1 + q^2$ which is circumscribed about the parabolid $2x = y^2 + z^2$ is the enveloping cylinder which touches it along its section by the plane $y + 1 = 0$.

19. Determine the solution of the equation $\frac{\partial^4 z}{\partial x^4}+\frac{\partial^2 z}{\partial y^2}=0$ $(-\infty < x < \infty,\ y \geq 0)$ satisfying the conditions :

(a) z and its partial derivatives tend to zero as $x \to \pm\infty$;

(b) $z = f(x),\ \partial z / \partial y = 0$ on $Y = 0$.

20. Solve the one-dimensional diffusion equation in the region $0 \leq x \leq \pi,\ t \geq 0$, when

(a) θ remains finite as $t \to \infty$;

(b) $\theta = 0$ if $x = 0$ or π, for all value of t;

(c) At $t = 0, \begin{cases} \theta = x & 0 \leq x \leq \frac{1}{2}\pi \\ \theta = \pi - x & \frac{1}{2}\pi \leq x \leq \pi \end{cases}$.

D–6469

Sub. Code
31131

DISTANCE EDUCATION

M.Sc.(Mathematics) DEGREE EXAMINATION,
DECEMBER 2024.

Third Semester

DIFFERENTIAL GEOMETRY

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours   Maximum : 75 marks

SECTION A — ($10 \times 2 = 20$ marks)

Answer ALL the questions.

1. Define principal normal.

2. Define osculating circle.

3. Define an involute.

4. What is pitch of the helix?

5. Define the tangential components.

6. State the characteristics property of geodesic.

7. Give the christoffel symbols of first and second kind.

8. Explain geodesic parallels.

9. What do you mean by osculating developable of the curve?

10. Explain Rodrigues formula.

SECTION B — ($5 \times 5 = 25$ marks)

Answer ALL the questions, choosing either (a) or (b).

11. (a) Find the equation of the osculating plane of the curve given by

$\vec{r} = \{a \sin u + b \cos u, \; a \cos u + b \sin u, \; c \sin u\}$.

Or

(b) Prove that the involutes of a circular helix are plane curves.

12. (a) Find the area of the anchor ring.

Or

(b) Determine the coefficients of the direction which makes an angle $\pi/2$ with the direction whose coefficients are (l, m).

13. (a) Find the surface of revolution which is isometric with a region of the right helicoids.

Or

(b) Enumerate the normal property of geodesic.

14. (a) Prove that every helix an a cylinder is a geodesic.

Or

(b) Prove that the curves of the family $\frac{v^3}{u^2}$ = constant are geodesics on a surface with metric $v^2 du^2 - 2uv\, du\, dv + 2v^2 dv^2 \;\; (u > 0, v > 0)$.

15. (a) State and prove the Meusnier's theorem.

Or

(b) Enumerate the following terms.

(i) Dupin's indicatrix

(ii) Characteristic line.

SECTION C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Obtain the curvature and torsion of the curve of intersection of the two quadric surfaces $ax^2 + by^2 + cz^2 = 1$ and $a'x^2 + b'y^2 + c'z^2 = 1$.

17. State and prove fundamental existence theorem for space curve.

18. Find the geodesics on a surface of revolution.

19. State and prove Gauss – Bonnet theorem.

20. Prove that a necessary and sufficient condition that a curve on a surface be a line of curvature is that the surface normals along the curve form a developable.

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D–6470

Sub. Code
31132

DISTANCE EDUCATION

M.Sc.(Mathematics) DEGREE EXAMINATION, DECEMBER 2024.

Third Semester

OPTIMIZATION TECHNIQUES

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

SECTION A — ($10 \times 2 = 20$ marks)

Answer ALL the questions.

1. Define a cut and the cut capacity in a network.
2. Define critical activity.
3. What types of problems can be solved by means of O.R. models?
4. Define the total float and the free float.
5. Define critically path.
6. What is meant by minimax value of the game?
7. Explain steepest ascent method.
8. Define stationary point.
9. Define general constrained non-linear programming problem.
10. Define quadratic programming model.

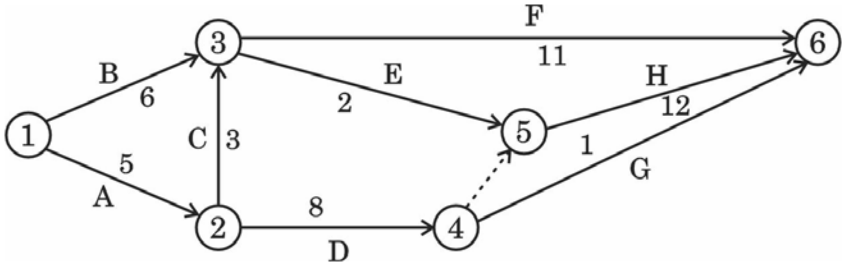
SECTION B — ($5 \times 5 = 25$ marks)

Answer ALL the questions, choosing either (a) or (b).

11. (a) Describe Dijkstra's algorithm.

Or

(b) Determine the critical path for the network.



All the durations are in days.

12. (a) Solve : Maximize $z = 2x_1 + x_2$ subject to the constraints $x_1 + 5x_2 \leq 10$, $x_1 + 3x_2 \geq 6$, $2x_1 + 2x_2 \leq 8$, $x_2 \geq 0$ and x_1 unrestricted.

Or

(b) Solve : Minimize $z = 3x_1 - 2x_2 + x_3$ subject to the constraints : $2x_1 - 3x_2 + x_3 \leq 5$, $4x_1 - 2x_2 \geq 9$, $-8x_1 + 4x_2 + 3x_3 = 8$; $x_1, x_2 \geq 0$ and x_3 unrestricted.

13. (a) Solve the following game and determine the value of the game

Player B

$$\text{Player A} \begin{pmatrix} 1 & 3 \\ 4 & 1 \end{pmatrix}$$

Or

(b) Solve the following game by simplex method

Player B

$$\text{Player A} \begin{pmatrix} 1 & -1 & 3 \\ 3 & 5 & -3 \\ 6 & 2 & -2 \end{pmatrix}$$

14. (a) Is the following two – person zero – sum game stable? (The payoff is for player A). Solve the game problem

Player B

$$\text{Player A} \begin{pmatrix} 8 & 6 & 2 & 8 \\ 8 & 9 & 4 & 5 \\ 7 & 5 & 3 & 5 \end{pmatrix}$$

Or

(b) Solve the LPP by Jacobian method :

Maximize : $f(X) = 5x_1 + 3x_2$

Subject to

$$g_1(X) = x_1 + 2x_2 + x_3 - 6 = 0$$
$$g_2(X) = 3x_1 + x_2 + x_4 - 9 = 0$$
$$x_1, x_2, x_3, x_4 \geq 0.$$

15. (a) Prove that a sufficient condition for a stationary point X_0 to be extremum is that the Hessian matrix H evaluated at X_0 is (i) Positive definite when X_0 is a minimum point (ii) negative definite when X_0 is a maximum point.

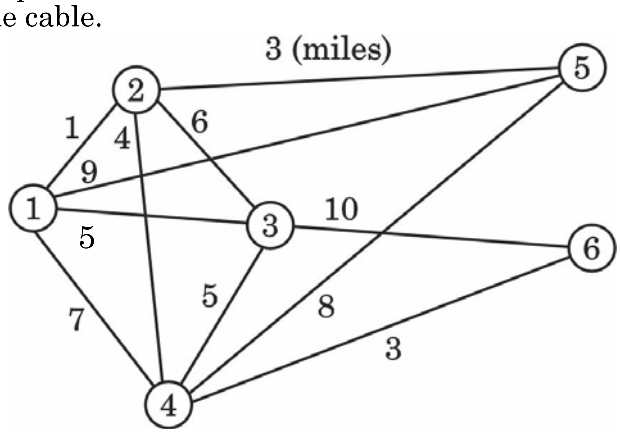
Or

(b) Discuss about separable programming.

SECTION C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Find the minimal spanning tree of the network under the independent conditions : Node 5 and 6 are linked by a 2-mile cable.



17. For the purpose of preparing its next year's budget, a company must gather information from its sales, production, accounting and treasury departments. The given table indicates the activities and their durations. Prepare the network model of the problem and carry out the critical path computation.

Activity	Description	Immediate predecessors (s)	Duration (days)
A	Forecast sales volume	–	10
B	Study competitive market	–	7
C	Design item and facilities	A	5
D	Prepare production schedules	C	3
E	Estimate cost of production	D	2
F	Set sales price	B, E	1
G	Prepare budget	E, F	14

18. Solve the problem by the revised simplex method :

Maximize $z = 2x_1 + x_2 + 2x_3$

Subject to

$$4x_1 + 3x_2 + 8x_3 \leq 12$$
$$4x_1 + x_2 + 12x_3 \leq 8$$
$$4x_1 - 2x_2 + 3x_3 \leq 8$$
$$x_1, x_2, x_3 \geq 0.$$

19. Apply the Newton – Raphson method to solve : $f(X) = 4x^4 - x^2 + 5$. Also examine for extreme points.

20. Write the Kuhn – Tucker necessary conditions for the following :

Minimize : $f(X) = x_1^4 + x_2^2 + 5x_1x_2x_3$

Subject to

$$x_1^2 - x_2^2 + x_3^2 \leq 10$$
$$x_1^3 + x_2^2 + 4x_3^2 \geq 20.$$

D–6471

Sub. Code
31133

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION,
DECEMBER 2024.

Third Semester

ANALYTIC NUMBER THEORY

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL the questions.

1. When will you say an integer is square free?

2. State Euclids lemma.

3. If $(a,b)=(a,c)=1$, prove that $(a,bc)=1$.

4. State the legendre identity.

5. Define the Riemann – Zeta function.

6. When will you say the functions are divisor functions?

7. If $a \equiv b (\bmod m)$ and $a \equiv b (\bmod n)$, where $(m,n)=1$, then prove that $a \equiv b (\bmod mn)$.

8. Prove that $[-x] = \begin{cases} [x] & if \quad x = [x] \\ -[x]-1 & if \quad x \neq [x] \end{cases}$.

9. State the Chinese remainder theorem.

10. Write down the Diophantine equations.

SECTION B — (5 × 5 = 25 marks)

Answer ALL the questions, choosing either (a) or (b).

11. (a) Prove that the are infinitely many prime numbers.

Or

(b) State and prove division algorithm.

12. (a) Define the Euler totient function $\phi(n)$. If $n \geq 1$, prove that $\phi(n) = \sum_{d/n} \mu(d)\left(\frac{n}{d}\right)$.

Or

(b) Prove that $n^4 + 4$ is composite if $n > 1$.

13. (a) If $x \geq 1$, prove that $\sum_{n>x} \frac{1}{n^s} = 0\left(x^{1-s}\right)$, if $s > 1$.

Or

(b) Prove that the set lattice points visible from the origin has density $\frac{6}{\pi^2}$.

14. (a) If a prime p does not divide a, the prove that $a^{-1} \equiv 1(\text{mod } p)$.

Or

(b) Prove that $5n^3 + 7n^2 \equiv 0(\text{mod} 12)$ for all integers n.

15. (a) Show that the Legendre's symbol (n/p) is completely multiplicative function of n.

Or

(b) If p is an odd positive integer, prove that

(i) $(-1/p) = (-1)^{(p-1)/2}$

(ii) $(2/p) = (-1)^{(p^2-1)/8}$.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. (a) Prove that if $2^n - 1$ is prime, then n is a prime.

(b) State and prove the fundamental theorem of arithmetic.

17. Define Liouville's function $\lambda(n)$, for every $n \geq 1$. Prove that $\sum_{d/n} \lambda(d) \begin{cases} 1, & \text{if } n \text{ is a square} \\ 0, & \text{otherwise} \end{cases}$. Also prove that $\lambda^{-1}(n) = |\mu(n)|$ for all n.

18. State and prove Eulers summation formula.

19. State and prove Wilson's theorem.

20. State and prove Gauss's lemma.

D–6472

Sub. Code
31134

DISTANCE EDUCATION

M.Sc.(Mathematics) DEGREE EXAMINATION,
DECEMBER 2024.

Third Semester

STOCHASTIC PROCESSES

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL the questions.

1. Define stochastic processes.

2. When do you say a Markov chain is homogenous?

3. Define states of a Markov chain.

4. Write the backward diffusion equation of the wiener process.

5. Define trajectories.

6. Is the Wiener process as covariance stationary? Justify.

7. State the Bienayame – Galton – Watson process.

8. Define a Markov renewal branching process.

9. Explain loss system.

10. State the Erlang's first formula.

SECTION B — (5 × 5 = 25 marks)

Answer ALL the questions, choosing either (a) or (b).

11. (a) Explain the random walk between two barriers.

Or

(b) Derive the Poisson law $P_n(t) = \frac{e^{-\lambda t}(\lambda t)^n}{n!}$, $n = 0, 1, 2, \ldots$

12. (a) For the Poisson process $\{N(t)\}$ as $t \to \infty$, prove that $P_r\left\{\left|\frac{N(t)}{t} - \lambda\right| \geq \in\right\} \to 0$ where $\in > 0$ is a preassigned number.

Or

(b) If $X(t)$ with $X(0)$ and $\mu = 0$ is a Wiener process and $0 < s < t$, show that for at least one λ satisfying $s \leq \tau \leq t$ $\Pr(X(\tau) = 0\} = \left(\frac{2}{\pi}\right)\cos^{-1}\left((s/t)^{1/2}\right)$.

13. (a) Prove that the probability of extinction q is the smallest root in [0, 1] of the equation $u(s) = 0$; further, $q = 1 (q < 1)$ if and only if $u'(1) \leq (>) 0$.

Or

(b) Show that the m.g.f. $\phi_n(s) = E\{e^{-sW_n}\}$, $n = 1, 2, \ldots$ satisfies the relation $\phi_{n+1}(ns) = P[\phi_n(s)]$.

14. (a) Prove that $P_n(s) = P_{n-1}(P(s))$ and $P_n(s) = P(P_{n-1}(s))$, where $P_n(s) = \sum_k P_k s^k$.

Or

(b) If $m = 1, \sigma^2 < \infty$, then prove that $\lim_{n \to \infty} n P_r\{X_n > 0\} = \frac{2}{\sigma^2}$.

15. (a) Explain about waiting time in queue.

Or

(b) Explain the model with finite input source : $[M / M / s / m]$.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Derive the Chapman – Kolmogorov equation.

17. Prove that, if the intervals between successive occurrences of an even E are independently distributed with a common exponential distribution with mean $1/\lambda$ then the events E form a Poisson process with mean λt.

18. Let $\{X(t), 0 \le t \le T\}$ be a Wiener process with $X(0) = 0$ and $\mu = 0$ Let $M(T) = \max_{0 \le t \le T} X(t)$. Then prove that for any $a > 0$, $\Pr\{M(T) \ge a\} = 2 \ \Pr\{X(T) \ge a\}$

19. State and prove Yaglom's theorem.

20. Derive the steady state probabilities $P_{1,n}, P_{0,q}$ and $P_{0,0}$ of the Poisson quene with general bulk service rule $[M / M(a,b) / 1 \text{ model}]$.

D–6473

Sub. Code
31141

DISTANCE EDUCATION

M.Sc.(Mathematics) DEGREE EXAMINATION,
DECEMBER 2024.

Fourth Semester

GRAPH THEORY

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL the questions.

1. Define isomorphism between two graphs with an example.

2. Draw the Petersen graph.

3. Define a walk and a cycle.

4. When will you say a graph is connected?

5. Find the Ramsey number $\gamma(1,1)$.

6. Define edge chromatic number.

7. State the four colour theorem.

8. Define plane graph and dual graph.

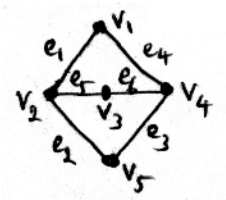
9. Define directed graph with an example.

10. Define Hamiltonian graph and Hamiltonian cycle.

SECTION B — ($5 \times 5 = 25$ marks)

Answer ALL the questions, choosing either (a) or (b).

11. (a) Define adjacency matrix of a graph. Also find the adjacency matrix of the graph :



Or

(b) Prove that G is a tree if and only if G is connected and every edge of G is a bridge.

12. (a) If G is a graph in which the degree of every vertex is atleast two then prove that G contains a cycle.

Or

(b) Show that a tree has at most one perfect matching.

13. (a) If G is K-critical, then prove that $\delta(G) \geq K-1$.

Or

(b) Prove that every 2-connected plane graph can be embedded in the plane so that any specified face is the exterior face.

14. (a) An edge e of a connected graph G is a bridge if and only if e is not on any cycle of G.

Or

(b) If G is planar, show that every subgraph of G is planar.

15. (a) Prove that in a digraph D, sum of the indegrees of all the vertices is equal to the sum of their out degrees, each sum being equal to the number of ares in D.

Or

(b) State and prove max-flow, min-cut theorem.

SECTION C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Prove that the maximum number of edges among all p vertex graphs with no triangle is $\left[\frac{p^2}{4}\right]$.

17. State and prove Chavatal theorem.

18. State and prove Vizing theorem.

19. Prove that every planar graph is 5-colourable.

20. Prove that a weak digraph D is Eulerian if and only if every vertex of D has equal indegree and outdegree.

D–6474

Sub. Code
31142

DISTANCE EDUCATION

M.Sc.(Mathematics) DEGREE EXAMINATION,
DECEMBER 2024.

Fourth Semester

FUNCTIONAL ANALYSIS

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

SECTION A — ($10 \times 2 = 20$ marks)

Answer ALL the questions.

1. Define a normed space with an example.

2. When do you say a normed space is a Banach space? Give an example.

3. Define bounded linear operator.

4. Define dual space.

5. Define orthonormal set. Give an example.

6. Define Hilbert space.

7. Define Annihilators.

8. What is meant by self-adjoint operator?

9. State the uniform boundedness theorem.

10. State the open mapping theorem.

SECTION B — ($5 \times 5 = 25$ marks)

Answer ALL the questions, choosing either (a) or (b).

11. (a) State and prove Jensen's inequality.

Or

(b) Show that l_p is separable for $1 \leq p < \infty$.

12. (a) Let X be a normed space and let Y be a subspace of X. If Y is finite dimensional, prove that Y is complete.

Or

(b) Prove that a Banach space cannot have a denumerable basis.

13. (a) If X is an inner product space, then prove that inner product $\langle x, y \rangle$ a is continuous function mapping $X \times X$ into F.

Or

(b) If x and y are any two vectors in an inner product space, then prove that $\|x+y\|^2 + \|x-y\|^2 = 2\|x\|^2 + 2\|y\|^2$.

14. (a) If X is a separable inner product space and A is any orthonormal set in X, then prove that A is countable.

Or

(b) Suppose $A = \{X_\alpha\}_{\alpha \in \wedge}$ is a complete orthonormal set in the Hilbert space X. Prove that $[\overline{A}] = X$.

15. (a) Define weak convergence. By an example show that weak convergence does not imply strong convergence.

Or

(b) If A is self adjoint and λ is an eigen value of A, then prove that λ is real.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. State and prove Hahn Banach separation theorem.

17. Let X be a normed space. Prove that the following are equivalent.

(a) Every closed and bounded subset of X is compact

(b) The subset $\{x \in X : \|x\| \leq 1\}$ of X is compact

(c) X is finite dimensional.

18. If A is completely continuous, then prove that its conjugate map A' is completely continuous.

19. State and prove Bessel's inequality.

20. State and prove closed graph theorem.

D–6475

Sub. Code
31143

DISTANCE EDUCATION

M.Sc.(Mathematics) DEGREE EXAMINATION,
DECEMBER 2024.

Fourth Semester

NUMERICAL ANALYSIS

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL the questions.

1. What is the order of convergence of iterative method?

2. State the strum theorem.

3. Write the deflated polynomial.

4. What is interpolation and extrapolation?

5. What is meant by truncation error?

6. State the Hermite interpolating polynomial.

7. Define spline function.

8. What is meant by error constant?

9. Write the formula for Taylor's series method.

10. When do you say the linear multistep method is conditionally stable?

SECTION B — ($5 \times 5 = 25$ marks)

Answer ALL the questions, choosing either (a) or (b).

11. (a) Obtain the complex roots of the equation $f(z) = z^3 + 1 = 0$. Use the initial approximation to a roots as $(x_0, y_0) = (0.25, 0.25)$.

Or

(b) Find all the roots of the equation $x^3 - 6x^2 + 11x - 6 = 0$ by Graeffe's method.

12. (a) Find the condition number of the system $\begin{bmatrix} 2.1 & 1.8 \\ 6.2 & 5.3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2.1 \\ 6.2 \end{bmatrix}$.

Or

(b) Let A be a square matrix. Then prove that $\lim_{m \to \infty} A^m = 0$ if $\|A\| < 1$ or if and only if $\delta(A) < 1$.

13. (a) Obtain the piecewise linear interpolating polynomials for the function $f(x)$ given below :

x	1	2	4	8
$f(x)$	3	7	21	73

Hence, estimate the values of $f(3)$ and $f(7)$.

Or

(b) Using linear interpolation, find $f(0.25, 0.75)$ for a function $f(x, y)$:

y \ x	0	1
0	1	1.414214
1	1.732051	2

14. (a) Obtain a linear polynomial approximation to the function $f(x) = x^3$ on the interval [0, 1] using the least squares approximation with $W(x) = 1$.

Or

(b) Define $S(h) = \frac{-y(x+2h) + 4y(x-h) - 3y(x)}{2h}$. Show that $y'(x) - S(h) = c_1 h^2 + c_2 h^3 + c_3 h^4 + \cdots$ and state c_1.

15. (a) Given the equation $y' = x + \sin y$ with $y(0) = 1$, show that it is sufficient to use Euler method with step size $h = 0.2$ to compute $y(0.2)$ with an error less than 0.05.

Or

(b) Apply the Taylor's series second order method to integrate $y' = 2t + 3y,\ y(0) = 1,\ t \in (0, 0.4)$ with $h = 0.1$.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Use synthetic division and perform two iterations of the Birge – Vieta method to find the smallest positive root of the equation $x^4 - 3x^3 + 3x^2 - 3x + 2 = 0$. Use the initial approximation $P_0 = 0.5$.

17. Find all the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 3 \end{bmatrix}$, by using Jacobi method.

18. Construct the Hermite interpolation polynomial that fits the data

x	$f(x)$	$f'(x)$
1	7.389	14.778
2	54.598	109.196

Estimate the value of $f(1.5)$.

19. Evaluate the integral $\int_0^1 \frac{dx}{1+x}$ using trapezoidal rule and Simpson's rule. With 2, 4 and 8 equal subintervals.

20. Using $R-K$ method of fourth order, find y for x = 0.1, x = 0.2, x = 0.3 given that $y' = xy + y^2$, $y(0) = 1$.

D–6476

Sub. Code
31144

DISTANCE EDUCATION

M.Sc.(Mathematics) DEGREE EXAMINATION,
DECEMBER 2024.

Fourth Semester

PROBABILITY AND STATISTICS

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL the questions.

1. Define random variable.

2. Define conditional p.d.f.

3. Let X have the p.d.f. $f(x) = \begin{cases} \frac{1}{5}, & 0 < x < 5 \\ 0, & \text{elsewhere} \end{cases}$. Determine $E(X)$.

4. Show that the random variables X_1 and X_2 with joint p.d.f. $f(x_1, x_2) = \begin{cases} x_1 + x_2, & 0 < x_1 < 1, 0 < x_2 < 1 \\ 0, & \text{elsewhere} \end{cases}$ are dependent.

5. Define negative Binomial distribution.

6. Define geometric distribution.

7. Let X have the p.d.f. $f(x)=\begin{cases}\frac{1}{3}, & x=1,2,3\\ 0, & \text{elsewhere}\end{cases}$. Find the p.d.f. of $Y=2X+1$.

8. Define convergence in distribution.

9. What is degenerate distribution?

10. Let Y be $b(72, 1/3)$. Find $\Pr(22 \le y \le 28)$.

SECTION B — (5 × 5 = 25 marks)

Answer ALL the questions, choosing either (a) or (b).

11. (a) Let X have the p.d.f. $f(x)=\begin{cases}2(1-x), & 0<x<1\\ 0, & \text{elsewhere}\end{cases}$, find $E(X)$, $E(X^2)$ and $E(6X+3X^2)$.

Or

(b) Find the mean and variance of the p.d.f. $f(x)=\begin{cases}6x(1-x), & 0<x<1\\ 0, & \text{elsewhere}\end{cases}$.

12. (a) Let X_1 and X_2 have the joint p.d.f. $f(x_1,x_2)=\begin{cases}2, & 0<x_1<x_2<1\\ 0, & \text{elsewhere}\end{cases}$. Find the marginal probability density function.

Or

(b) Let $f(x_1,x_2)=\begin{cases}4x_1x_2, & 0<x_1<1,\ 0<x_2<1\\ 0, & \textit{elsewhere}\end{cases}$ be a p.d.f. of X_1 and X_2. Find $\Pr(0<X_1<1/2,\ 1/4<X_2<1)$.

13. (a) Determine the mean and variance of gamma distribution.

Or

(b) Let X have the uniform distribution over the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Show that $y = \tan x$ has a Cauchy distribution.

14. (a) Let X and Y be random variables $\mu_1 = 1$, $\mu_2 = 4, \sigma_1^2 = 4$, $\sigma_2^2 = 6, \delta = \frac{1}{2}$. Find the mean and variance of $Z = 3X - 2Y$.

Or

(b) Let X be $n(\mu, \sigma^2)$ so that $\Pr(X < 89) = 0.90$ and $\Pr(X < 94) = 0.95$. Find μ and σ^2.

15. (a) If X_n have a gamma distribution with parameter $\alpha' = n$ and β, where β is not a function of n. Find the limiting distribution of $Y_n = \frac{X_n}{n}$.

Or

(b) Let Z_n be $\chi^2(n)$. Then prove that the random variable $Y_n = (z_n - n)/\sqrt{2n}$ has a limiting standard normal distribution.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. State and prove Chebyshev's inequality.

17. Let X_1 and X_2 denote the random variables that have joint p.d.f. $f(x_1,x_2)$, let $m(t_1,t_2)$ denote the m.g.f. of the distribution, show that X_1 and X_2 are independent if and only if $\mu(t_1,t_2)=\mu(t,0)\mu(0,t_2)$.

18. Find the m.g.f. of a normal distribution and hence find the mean and variance of the normal distribution.

19. Derive the p.d.f. of $F-$ distribution.

20. State and prove the central limit theorem.

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